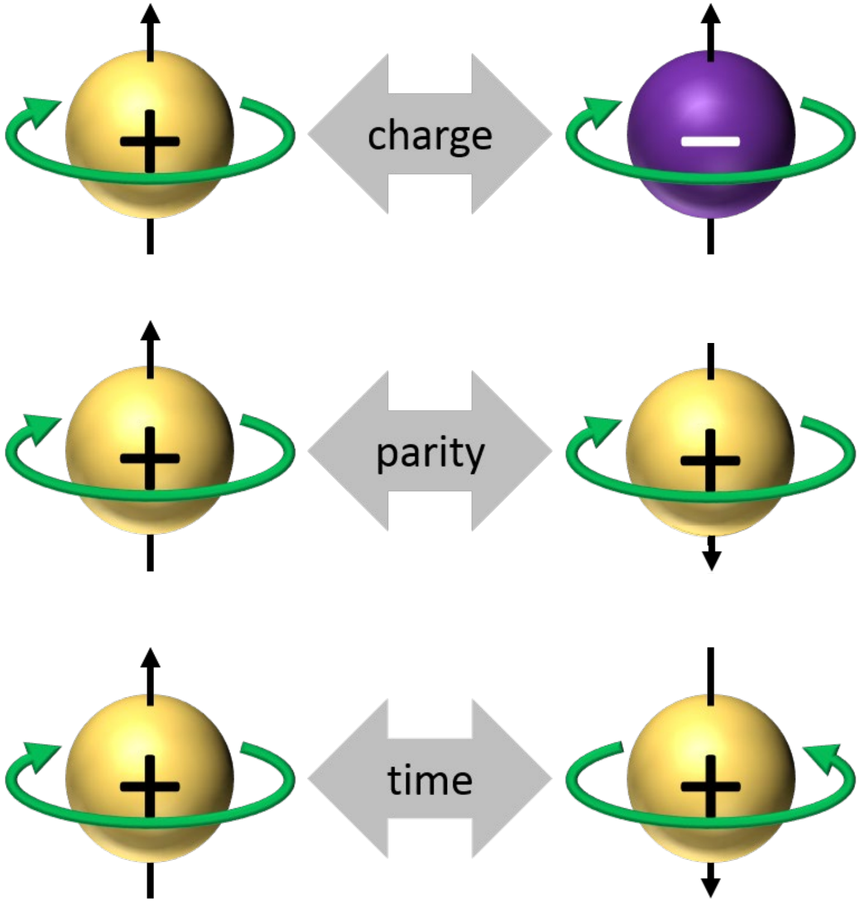
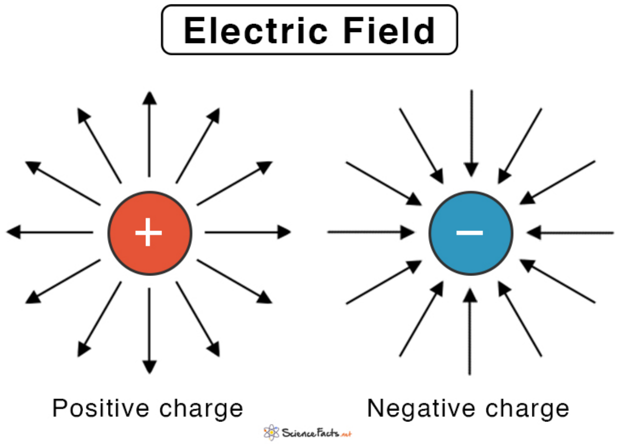
The Discrete Fundamental Symmetries

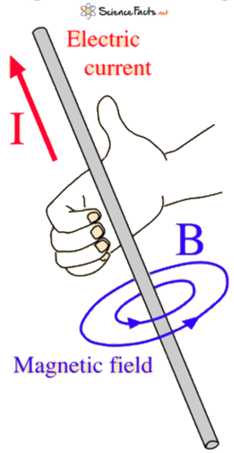
- Charge-conjugation (C): replace matter with antimatter
  - Charge changes sign. Polar vectors (e.g. momentum) and axial-vectors (e.g. spin) are unchanged.
- Parity (P): spatial reflection
  - Polar vectors change sign. Charge and axial-vectors are unchanged.
- Time (T): direction of the clock
  - Axial-vectors change sign. Charge and polar vectors are unchanged.

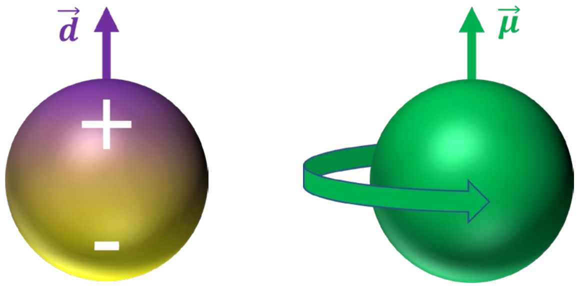


Homework

- Review the definition of polar vectors and axial vectors. Is an electric field a polar vector or axial vector? Which is the magnetic field? How do these transform under C-symmetry, P-symmetry, and T-symmetry?
- A magnetic dipole moment $\vec{\mu}$ can be thought of as the strength of a current loop. How does it transform under C-symmetry, P-symmetry, and T-symmetry? Consider a particle with both a spin and a magnetic dipole moment. Does the initial state look like the final state after a T transformation? That is, does the magnetic dipole moment violate T symmetry?
- An electric dipole moment $\vec{d}$ measures a separation of charge, and points from negative to positive. How does it transform under C-symmetry, P-symmetry, and T-symmetry? Consider a particle with both a spin and an electric dipole moment. Does the initial state look like the final state after a T transformation? That is, does the electric dipole moment violate T symmetry?



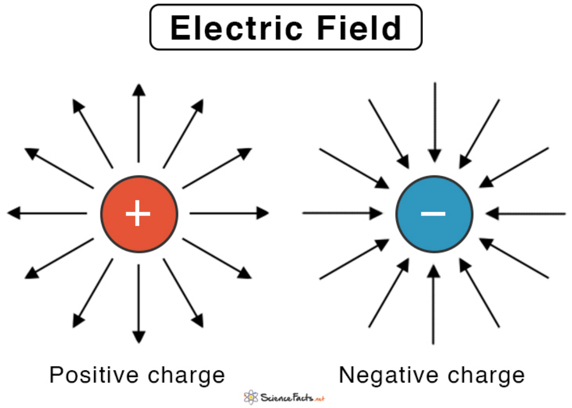


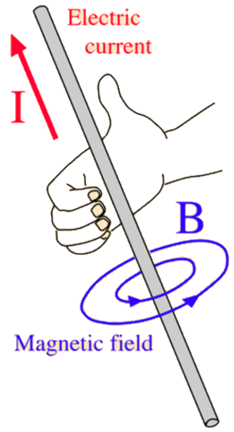


Electric dipole moment　Magnetic dipole moment

Answer #1

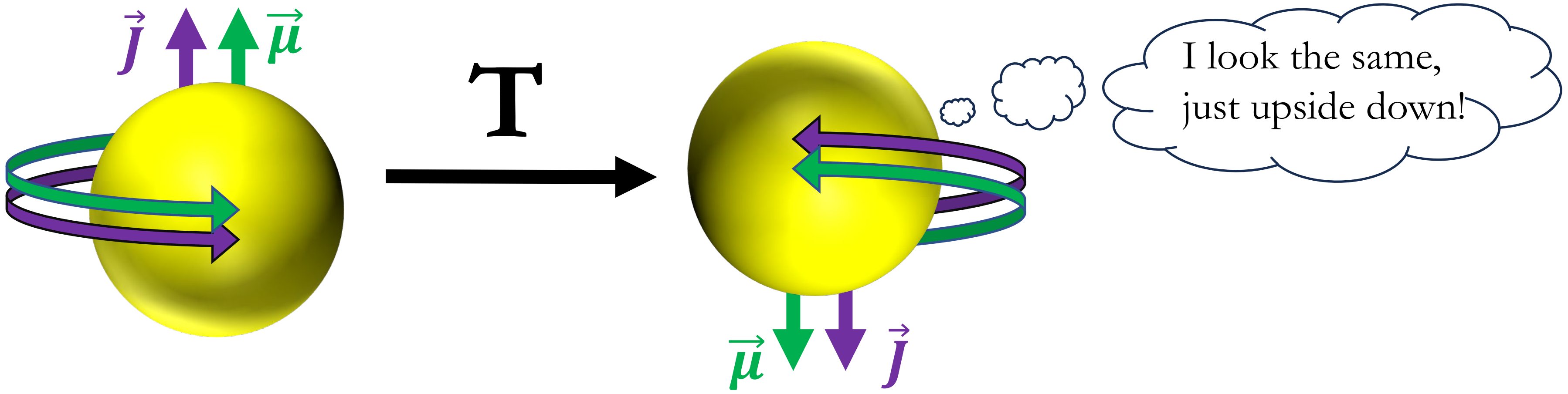
- Review the definition of polar vectors and axial vectors. Is an electric field a polar vector or axial vector? Which is the magnetic field? How do these transform under C-symmetry, P-symmetry, and T-symmetry?
  - Polar (ordinary) vectors can be thought of as an arrow. For example, a momentum vector is an arrow pointing in the direction of motion. Axial (or pseudo) vectors are used to describe rotational motion (for example, they can be built mathematically from cross-products of vectors) and point according to the right-hand rule. These are subtle concepts, and you may find it useful to read more explanations online after this exercise.
  - An electric field **points** away from or towards a position or negative charge, respectively, and is a polar vector. A magnetic field **curls** around a straight-line electric current, or points away from a **rotating** current according to the right-hand rule, and is an axial vector.
  - Under a C transformation, vectors and polar vectors are unchanged, according to slide 1. However (trick question!) the electric field is also proportional to charge, and **reverses** if a positive charge becomes a negative charge. Similarly, magnetic field is also proportional to current. Current is defined as the direction of positive charge, so if the charge changes sign, by definition the current changes direction, and therefore the magnetic field also **reverses**.
  - According to Slide 1, because the electric field is a polar vector, it **changes sign** under a P-transformation, and is **unchanged** under a T-transformation. Conversely, the magnetic field is an axial vector, so it is **unchanged** under a P-transformation, and **changes sign** under a T-transformation.





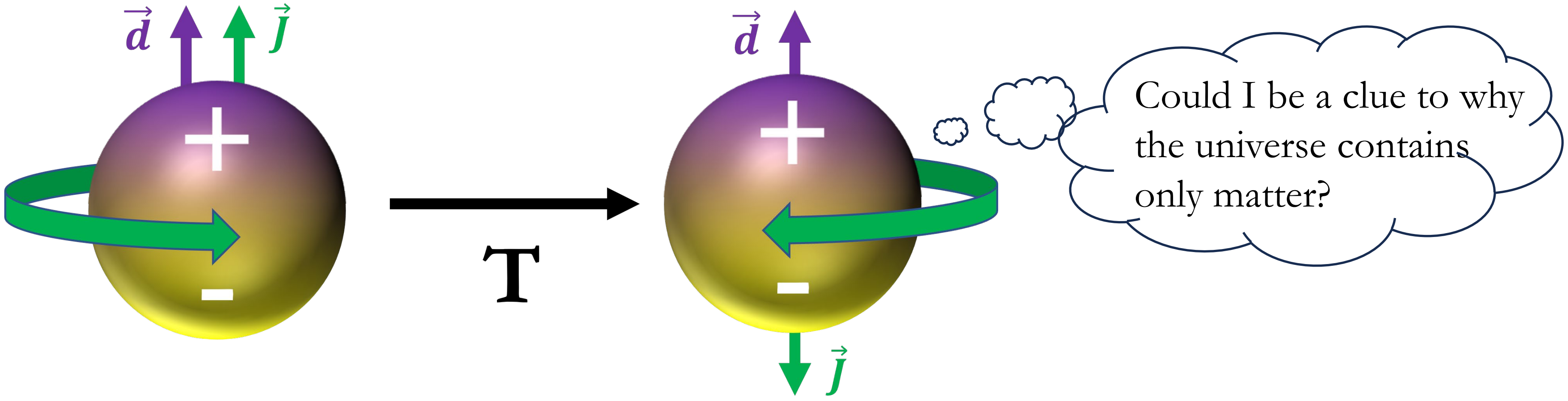
Answer #2

- A magnetic dipole moment $\vec{\mu}$ can be thought of as the strength of a current loop. How does it transform under C-symmetry, P-symmetry, and T-symmetry? Consider a particle with both a spin and a magnetic dipole moment. Does the initial state look like the final state after a T transformation? That is, does the magnetic dipole moment violate T symmetry?
  - A magnetic dipole moment of a particle is a fundamental quantum mechanical property, but we can think of it in a classical sense as originating from a current loop, similar to a magnetic field. It is an axial-vector and transforms **just like** the magnetic field does.
  - Spin is also an axial-vector. Both the magnetic dipole moment $\vec{\mu}$ and spin $\vec{J}$ transform the same way when T is reflected. You can think of it as the rotation moving backwards, so by the right-hand-rule, your thumb is now pointing down in both cases. The particle before and after the transformation is **indistinguishable**. T is therefore **not violated**. We say it is "conserved."



Answer #3

- An electric dipole moment $\vec{d}$ measures a separation of charge, and points from negative to positive. How does it transform under C-symmetry, P-symmetry, and T-symmetry? Consider a particle with both a spin and an electric dipole moment. Does the initial state look like the final state after a T transformation? That is, does the electric dipole moment violate T symmetry?
  - An electric dipole moment of a particle is a fundamental quantum mechanical property, but we can think of it as pointing from the negative "pole" to the positive "pole" of a particle. If charge is reversed, it also **reverses**. Otherwise, it is an ordinary pointing vector and transforms like a polar vector.
  - The electric dipole moment is a polar vector and is **unchanged** when time is reversed, unlike the spin. You **can distinguish** between a particle moving forward in time and one moving backwards in time, so T is **violated**.



Conclusion

- T-violation is one of the ingredients needed to create a matter-antimatter balance in the universe, and is a major open question. Scientists are looking for possibilities (like electric dipole moments) where it might be hiding.
- By learning how to classify different quantities according to their symmetry properties, we can simplify problems or even find new ways to test our understanding of the universe.
- Where can you use symmetry in your work and in your everyday life?