

Calibrating the Liquid Drop Model Challenge

<https://github.com/ascsn/theory-challenges>

The purpose of this challenge is for you to calibrate the Liquid Drop Model https://en.wikipedia.org/wiki/Semi-empirical_mass_formula.

If you have never done anything with Python, we suggest you take a look at this: https://www.youtube.com/watch?v=AJFen_Z3mWM&t=1524s. Also, ChatGPT can be of much help to start learning how to code well: <https://chatgpt.com/>

Your task are to:

*Non-Bayesian way:

- Import the data from the AME 2016 table (included in the github). We are only using nuclei above $A=16$ to avoid light nuclei where the LDM fails particularly. Perform a curve fit using the built in functions from python (https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.curve_fit.html) and take note of the reported uncertainties in the parameters.
- Construct a cost function as the sum of the squares of the residuals between your model predictions and the experimental data.
- Numerically minimize this cost function as a function of the four Liquid Drop Model parameters (search on google for scipy minimize function). The optimal parameters will come out of the minimization.
- Make a plot of the residual of your calibrated model and the experimental data. Notice anything interesting pattern?

*Bayesian way:

- Make a model calibration using the Bayesian formalism that is defined in the accompanying file "# Guided Example Bayesian calibration". For the error, use your estimation from the previous point (the model error in this case is much smaller than the actual experimental uncertainties).
- Plot the corner plot posterior as well as the model values on the Binding Energy per nucleon for the Calcium chain up to ^{60}Ca including the available experimental data.
- What would be the results if you have used in the calibration the Binding Energy per nucleon instead of the total Binding Energy?
- Bonus: Find the experimental error in the masses and repeat the calibration using only

experimental errors. This should give a good demonstration on the dangers of not taking into account model errors.

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In [2]: import numpy as np
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In [3]: data = np.loadtxt('Masses2016.txt', skiprows=1)

def LDM(params,x):
    #x = (n,z)
    #params= parameters (volume, surface, asymmetry, Coulomb)

    n=x[0]
    z=x[1]

    return params[0]*(n+z) - params[1]*(n+z)**(2/3) - params[2]*((n-z)**2/(n+z)) -
```

NuPEERS High Energy Physics Homework
Neutrinos are a thing
Dillard University
June, 2024

Neutrinos are a thing that oscillate in space in time as a function of distance in energy.

1. Starting with independent mass and flavor basis states of only two dimensions, derive the lepton mixing matrix and demonstrate the length and energy dependence of the oscillatory effect. Show the survivability of muon neutrinos as a function of length and energy.
2. Take what you learned above and expand to a three-dimensional lepton mixing matrix. Please define the Jarlskog invariant for leptons and show that a degeneracy exists for the mass splittings. Also show that if any of the mass splittings vanish, then the Jarlskog invariant also vanishes. Finally, derive the probability for the survivability of muon-neutrinos as a function of length and energy.
3. Try to expand this analysis to a four-dimensional lepton mixing matrix. Attempt to define the appearance probability for neutrinos of the fourth type from muon neutrinos. These are called sterile neutrinos. Derive the probability for the survivability of muon-neutrinos as a function of length, energy, and the magnitude of the fourth mass splitting.
4. Plot all muon-neutrino survivability probabilities as a function of L/E across 8 orders of magnitude starting at 10^{-3} for three different possible values for the mass splittings of the fourth neutrino state versus the first. Those values should be 5, 0.5, and 0.05 eV^2 . From this plot make a statement about the best possible location to distinguish which model exists in reality.