NuPEERS Lecture
Electron Scattering and the Nuclear Matter

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Who Am I?

Once Upon a Time
… the (real) twins

Second Upon a Time
… the (virtual) twins

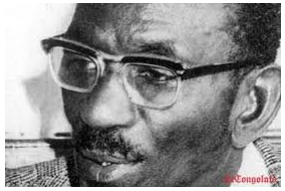
- **Influence**
  - Mame (grand-mother)
  - Wife: Mornetka
  - Daughter: Yannick (December 8)
- **Twin brother**: René
  - Son: Yoann (!)
  - Daughter: Maeva (December 8!)
- **Whole family …**

Education …

- **High School**
  - Lycée Lamine Gueye
- **College**
  - Université Cheikh Anta Diop (UCAD)
    - » BS: Physics & Chemistry (1987)
    - » MS: Physics & Chemistry (1990)
  - Université de Clermont-ferrand II (France)
    - » PhD: Nuclear Physics (1994)
- Positions
  - Hampton University/Jefferson Lab (VA)
    - » Researcher; Professor; Chair of Physics
  - Michigan State University (MI)

Cheikh Anta Diop
(Student of Pierre Joliot-Curie)

- 1923-1986
- 7 PhDs!
- ^{14}C (dating) lab

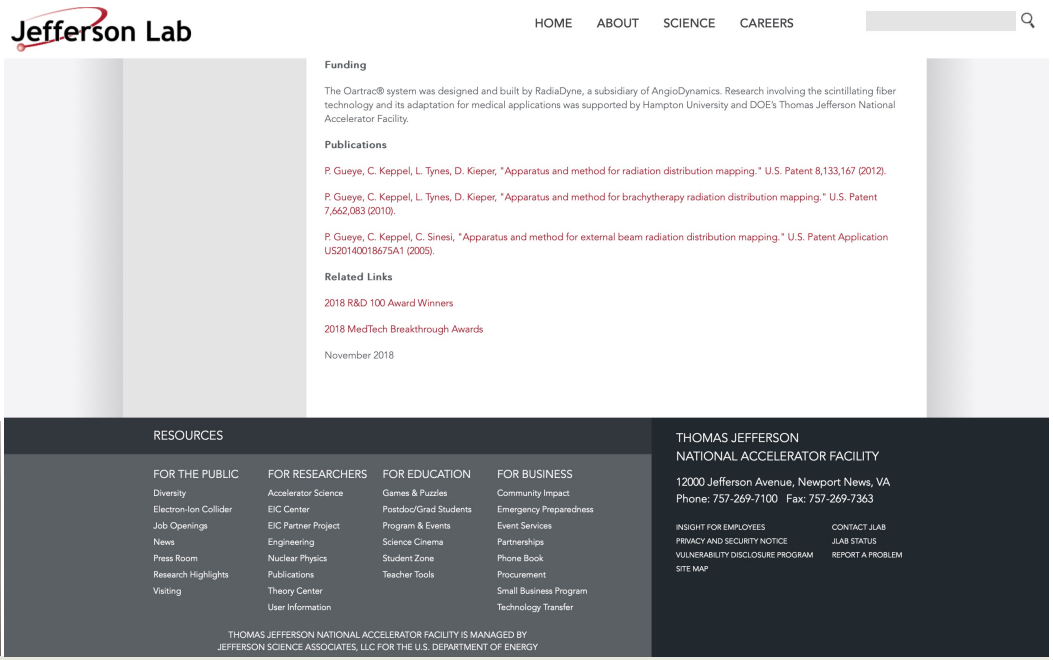


Research Interest [1]

- **Nuclear Physics**
  - Correction to the Born Approximation
  - Jefferson Lab (4-6-12 GeV era) – 1995+
  - Michigan State University: NSCL/FRIB (MoNA Collaboration) – 2012+
- **Accelerator Physics**
  - Beamline instrumentation: from beam to physics to patients
  - Low energy electron accelerator (LELIA) @ HU
  - US Particle Accelerator School
  - Polarized $e^{\pm}$ scattering off rare isotopes (https://indico.jlab.org/event/680)
- **Ultrafast Lasers (high power)**
  - University of Michigan (Gérard Mourou, 1997+ - 2018 Nobel Laureate)
    - » K. Assamagan *et al*, NIM, **A438** (1999)
  - High energy electrons from laser wakefield acceleration

Research Interest [2]

- **Medical Physics**
  - https://www.jlab.org/conference/amc-dps-22
  - Improving cancer treatments
    - » Scintillating fibers, 2001+
  - Brachytherapy (Curie therapy),
  - Proton therapy (HU Proton Therapy Institute)
  - Head Medical Physics Program 2011-2012
  - Patents:
    - » 9351691, 9028390, 8568285, 8133167, 7662083
  - G4DNA: Geant4 radiation damages to the DNA
    - » physics+biology+chemistry



Real time dosimetry monitoring for radiotherapy treatment

Developed by Jefferson Lab and Hampton University scientists; licensed to Radiadyne; R&D100 award, MedTech Breakthrough award

Active Catheter Development for Brachytherapy

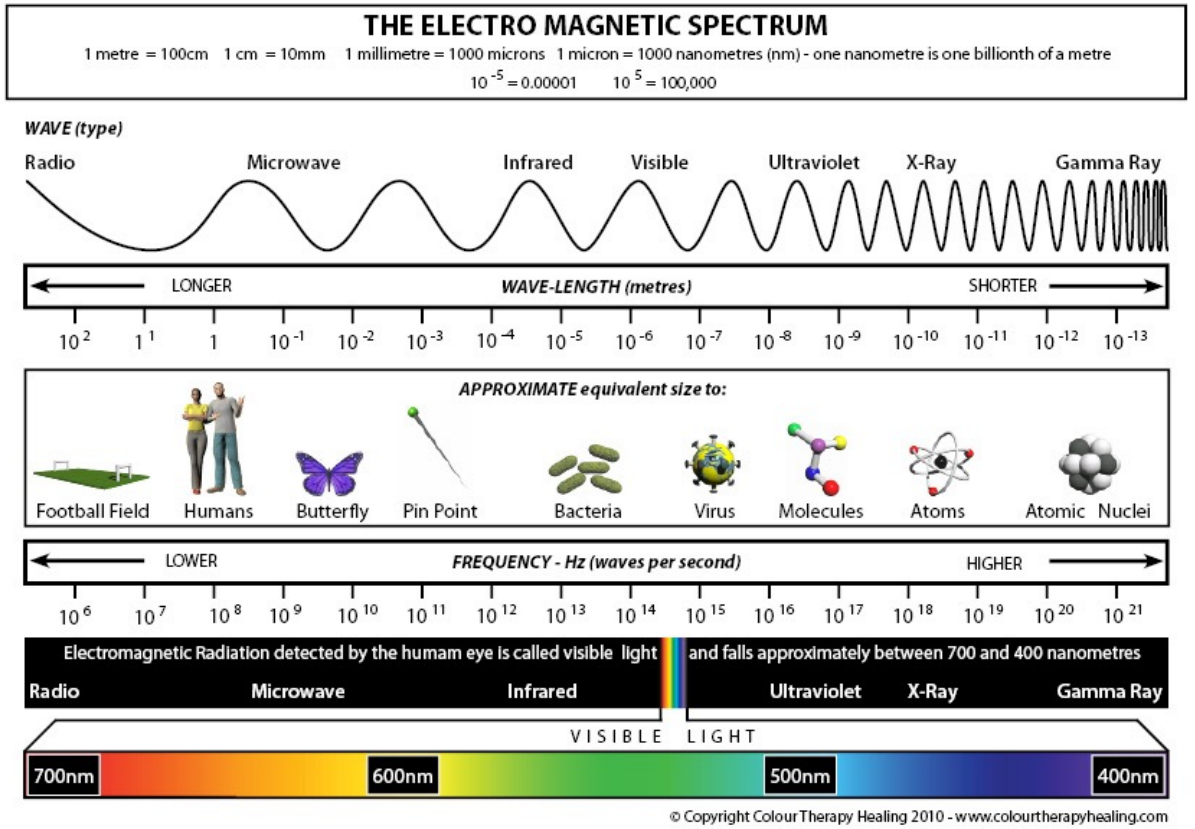
In-vivo dose measurements would allow for real time evaluation of dose delivered to patient

How Good Is Our Eyesight?

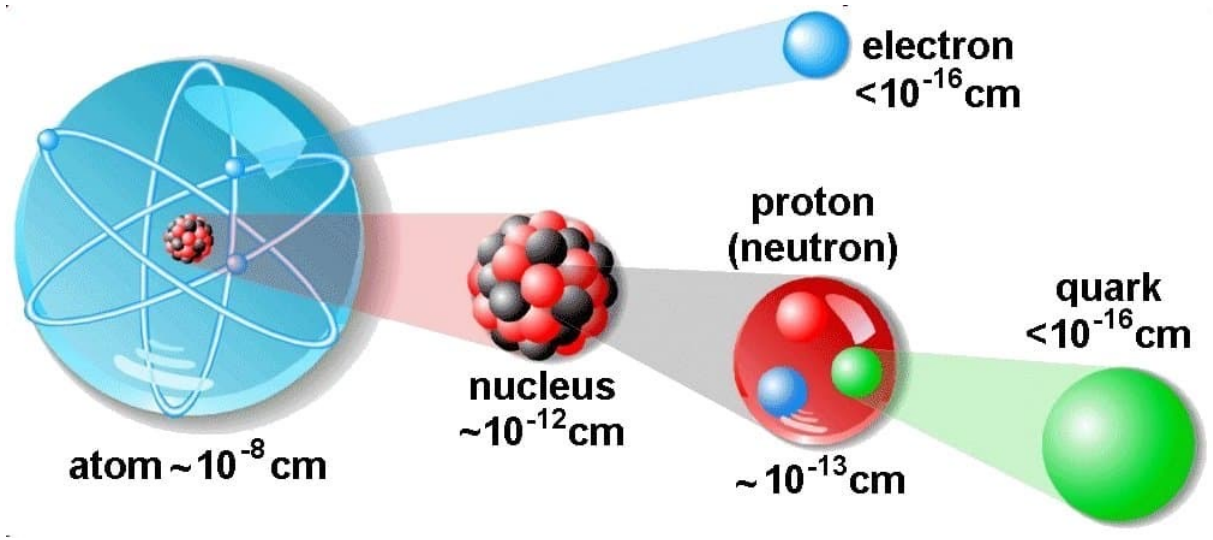
$E = hc/\lambda$

$hc \approx 200$ MeV fm
$\lambda \approx 2$ fm → $E \approx 100$ MeV

1 fm = 10^{-15} m!

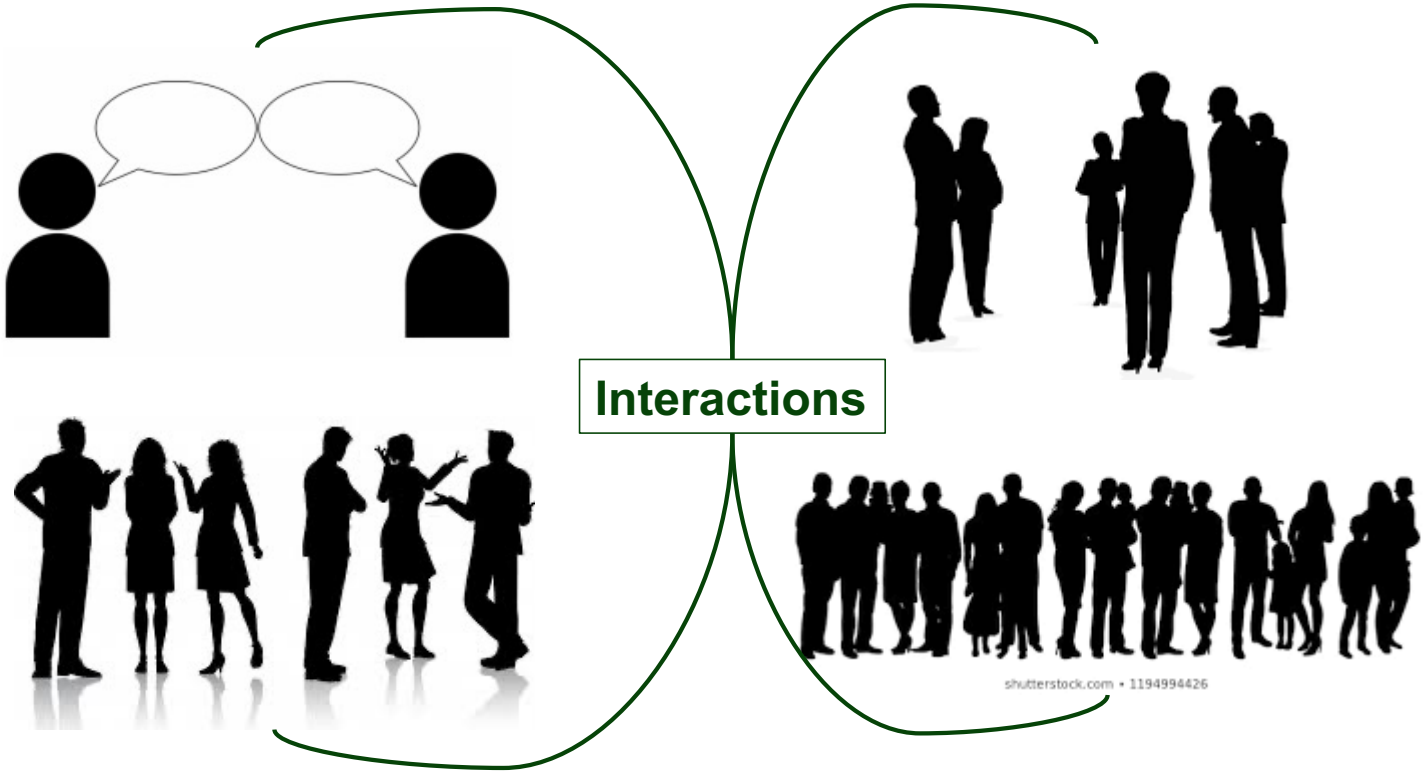


Making Atoms …

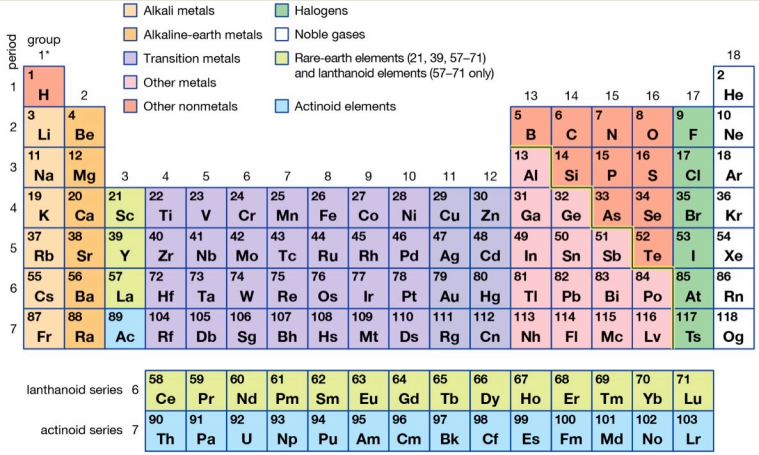


periodictable.org

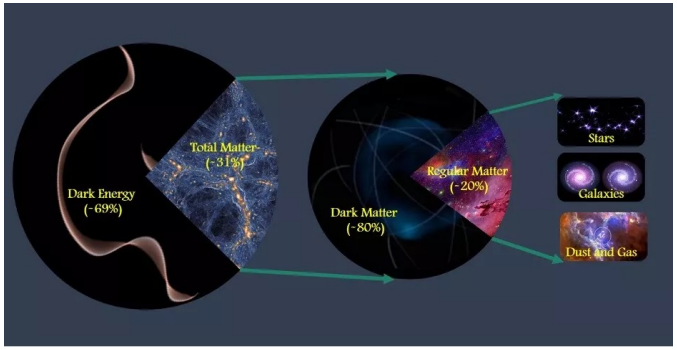
2, 3, 4 ... Too Many

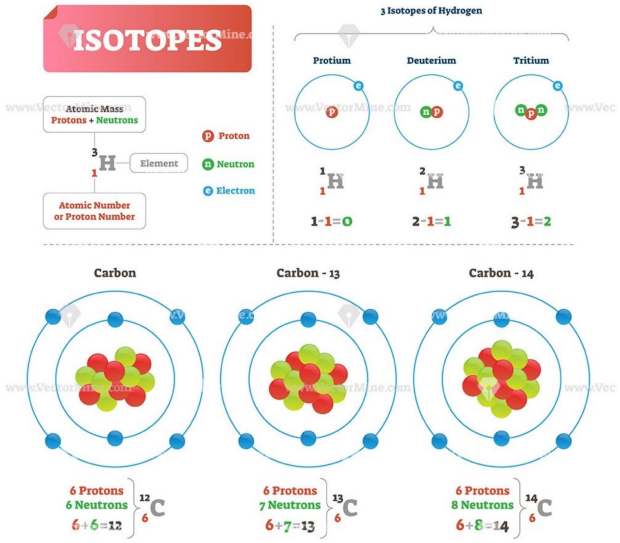


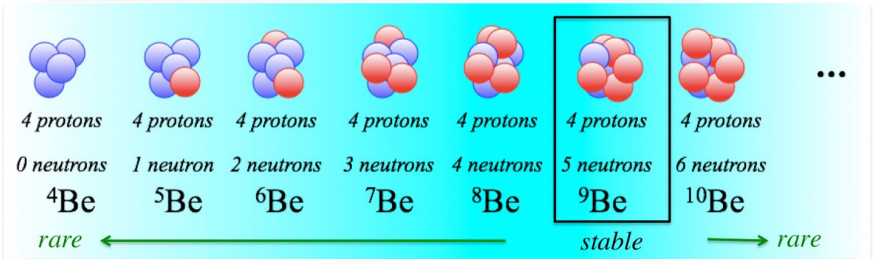
Making More Atoms …



https://www.space.com/universe-total-amount-matter-measured

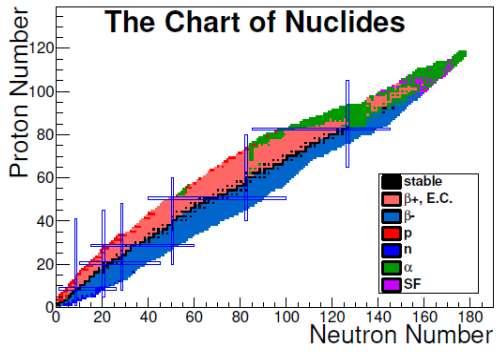






Nuclear Chart & Nuclear Science

Data from G. Audi, *et al.*, Chinese Physics C, 41(3),030001, 2017



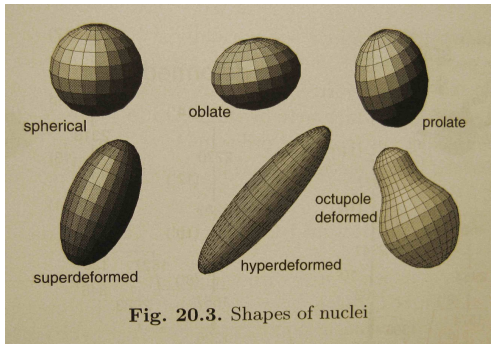
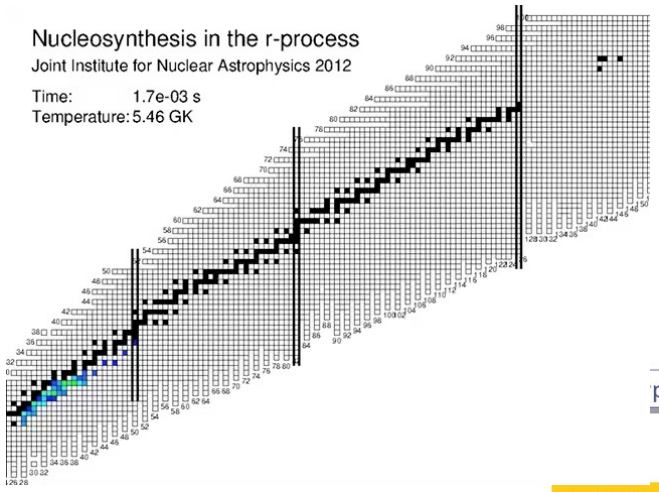
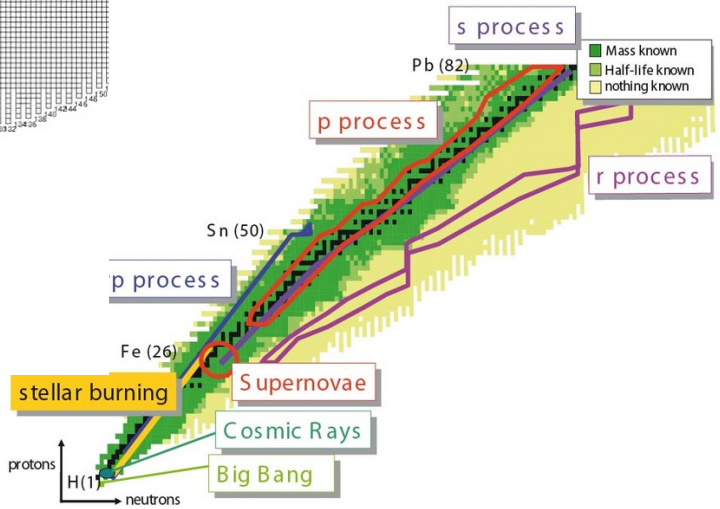


Fig. 20.3. Shapes of nuclei

https://web2.ph.utexas.edu/~coker2/index.files/rotation2.htm

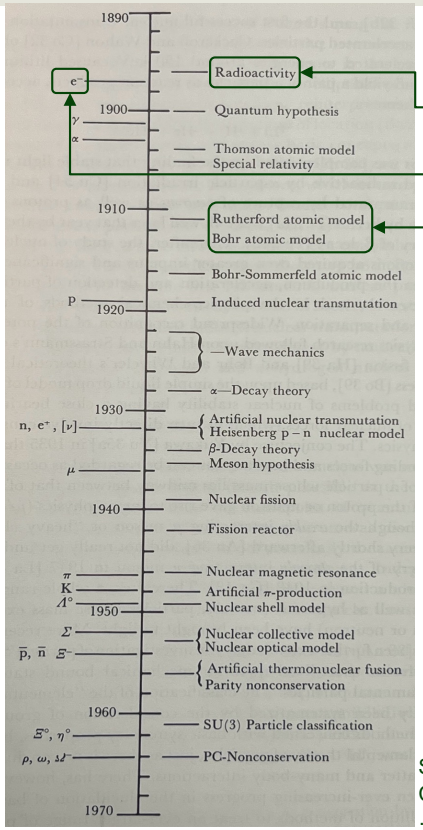
- 288 stables ($T_{1/2}$>life of the solar system)
- 3,308 different isotopes have been discovered (2019)





K.Blaum et al., Contemporary Physics, **51**, 149-175 2010

Nuclear Physics – Brief History



Physics of Nuclei and Particles
P. Marmier and E. Sheldon (1969)

Röntgen (1895)

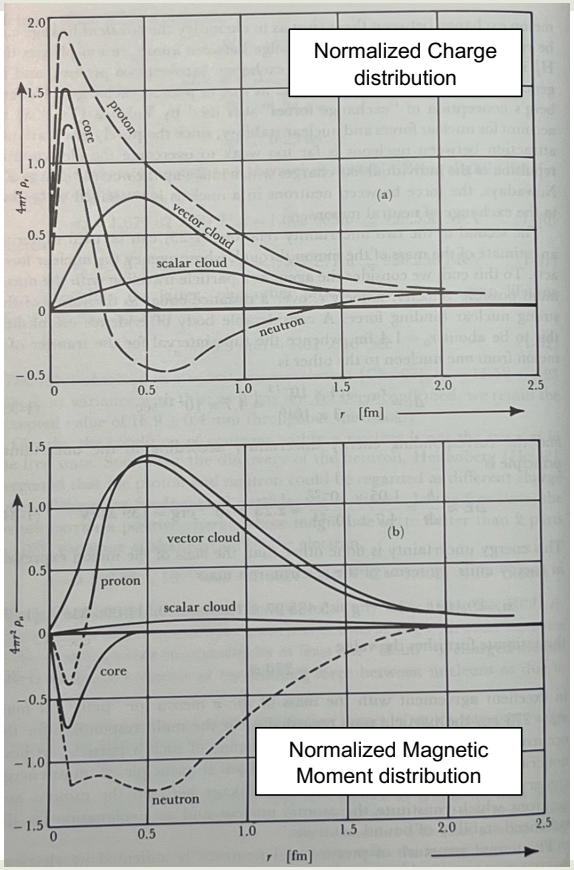
Thomson (1897)

Rutherford (1911)
Bohr (1912)

SLAC (20 GeV, e-)
CERN (28 GeV, p)
……

1990s
FRIB (200 MeV/u, ^{A}X)
……

Present → ?



Peaking Inside Closed Rooms

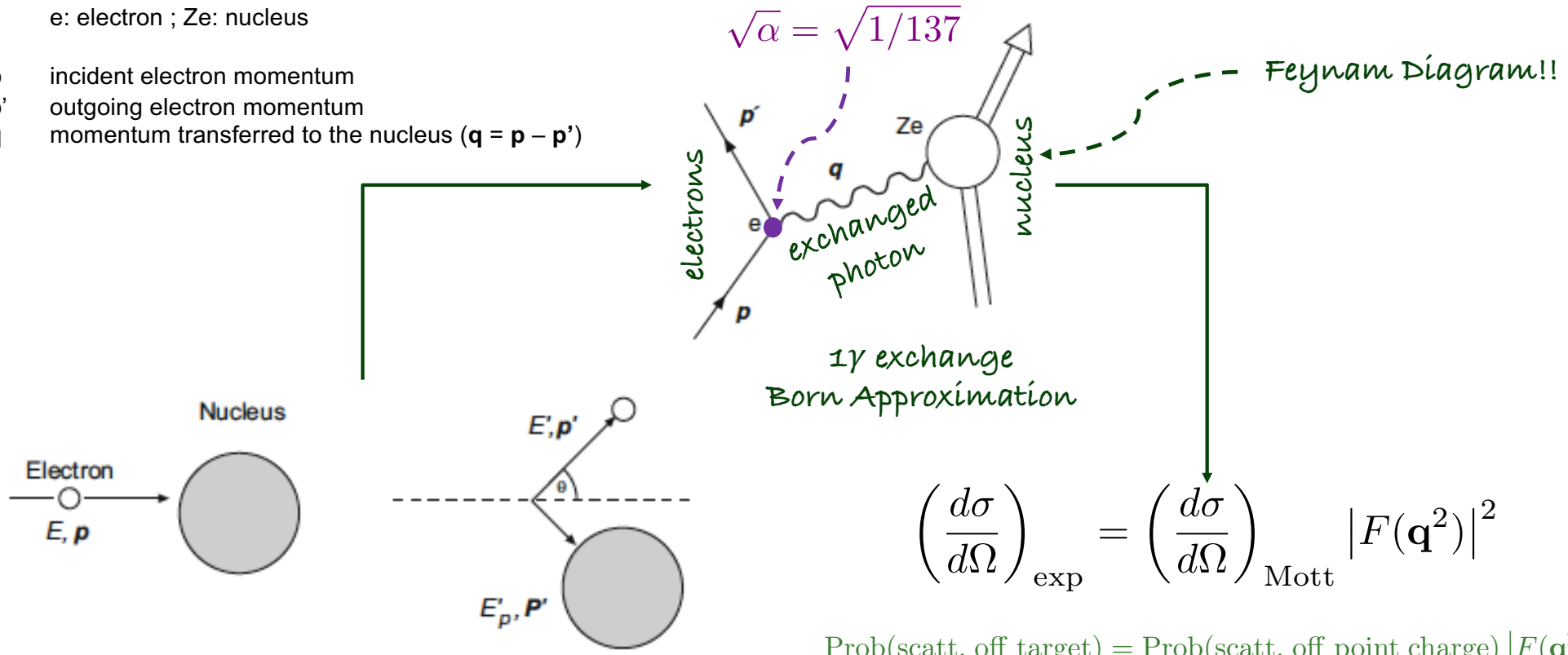
Can you really tell what is inside?

https://www.vecteezy.com/free-vector/house-cross-section

Peaking Inside Nuclei

e: electron ; Ze: nucleus

p incident electron momentum
p' outgoing electron momentum
q momentum transferred to the nucleus (**q** = **p** – **p'**)



$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left|F(\mathbf{q}^2)\right|^2$$

$$\text{Prob(scatt. off target)} = \text{Prob(scatt. off point charge)} \left|F(\mathbf{q}^2)\right|^2$$

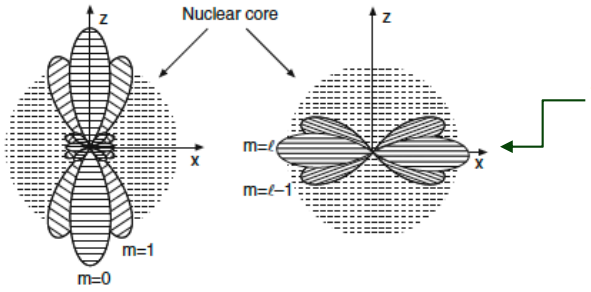
Nuclei: Spherical vs. Deformed (see C. Santamaria & P. Giuliani lectures)

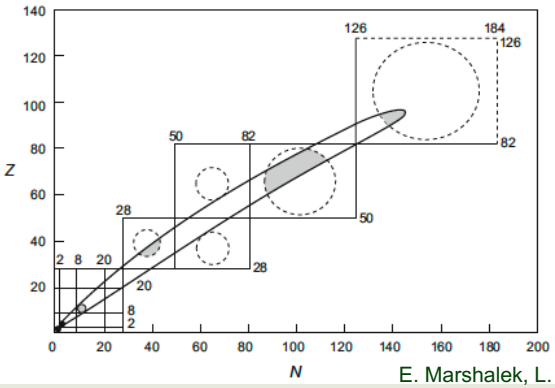
- Where are protons & neutrons?
  - Shell model
- What about nuclear shapes
  - Quadrupole moment: how deformed are nuclei?

$$Q_{class} = \int (3z^2 - \mathbf{x}^2)\rho(\mathbf{x})d^3x \qquad \langle R\rangle = (ab^2)^{1/3}; \Delta R = a - b$$

$$Q_{ellipsoid} = \frac{2}{5}Ze(a^2 - b^2) = \frac{6}{5}Ze\langle R\rangle^2\varepsilon \qquad \varepsilon = \frac{2}{3}\frac{\Delta R}{\langle R\rangle}$$

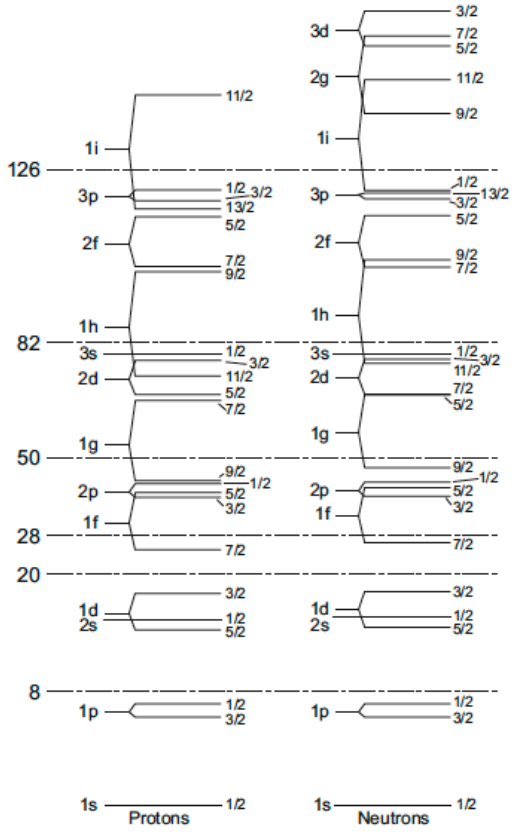
$$Q_{red} = \frac{Q_{ellipsoid}}{Ze\langle R\rangle^2}$$





E. Marshalek, L. Person, R. Sheline
Rev. Mod. Phys. 35, 108 (1963)

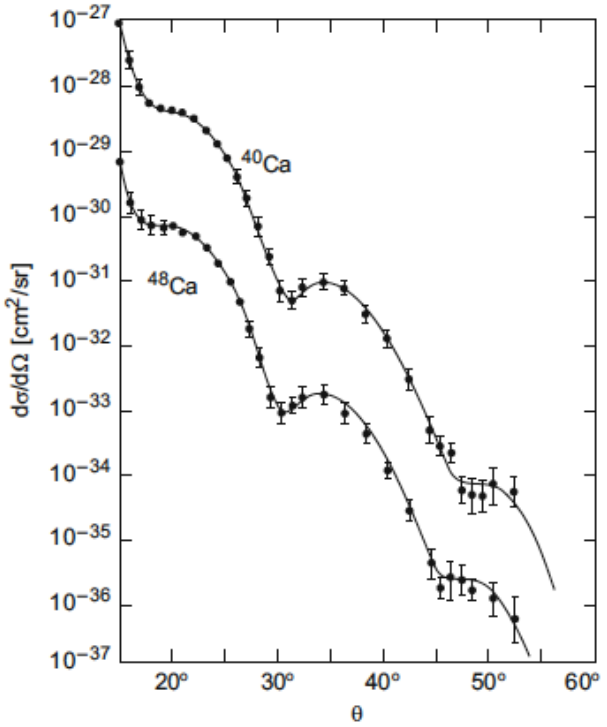
P.F.A. Klingenberg, Rev. Mod. Phys. 24, 63 (1952)



Most Important Quantity: The Nuclear Radius!

ρ(r)	$\|F(q^2)\|$	Example
pointlike	constant	Electron
exponential	dipole	Proton
gauss	gauss	^{6}Li
homogeneous sphere	oscillating	–
sphere with a diffuse surface	oscillating	^{40}Ca

r ⟶ $|q|$ ⟶

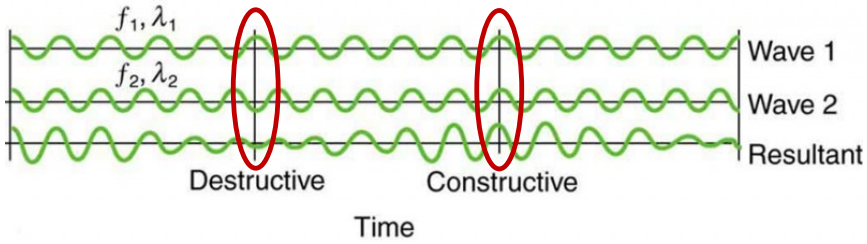


Bogdan Povh, Klaus Rith, Christoph Scholz, Frank Zetsche • Werner Rodejohann
Particles and Nuclei: An Introduction to the Physical Concepts

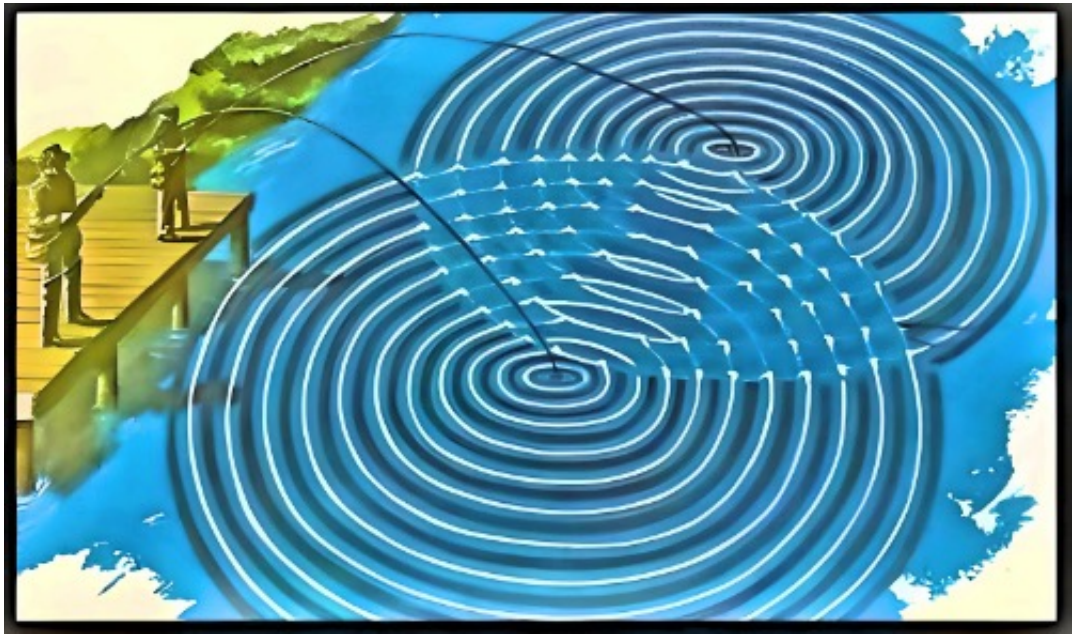
Waves Interference

- Microscopic objects
  - Electrons: point particles
  - Nucleons: ~1 fm (1 x 10^{-15} m)
- World of probability!!
  - Quantum mechanics
- Everything is energy = wave

$$E = mc^2$$

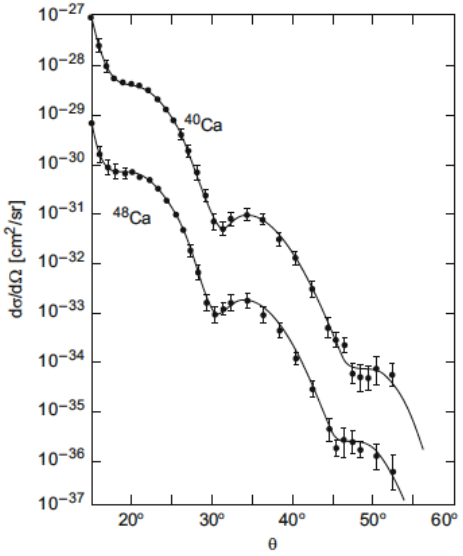


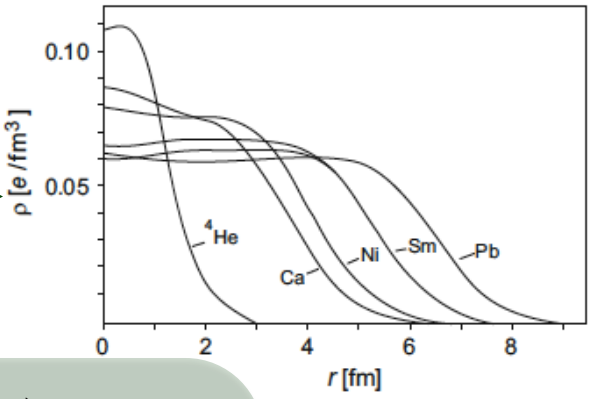
https://courses.lumenlearning.com/suny-physics/chapter/16-10-superposition-and-interference/



https://studymind.co.uk/notes/interference/

Extracting Nuclear Radii

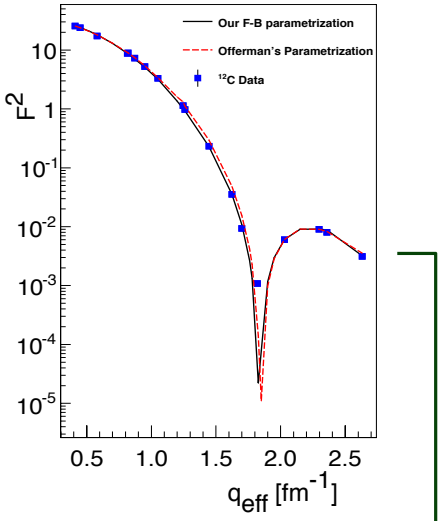


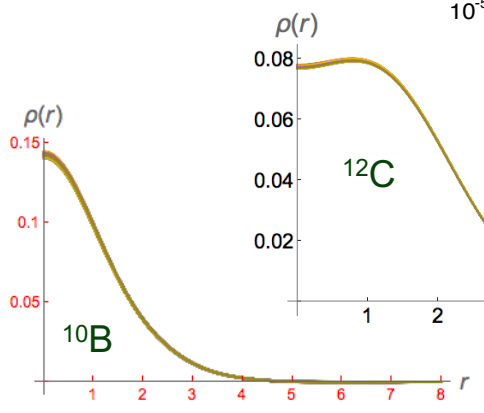


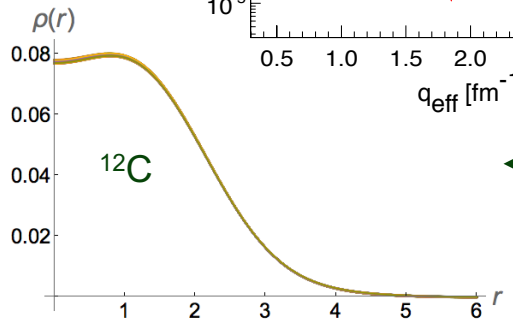
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F(\mathbf{q}^2)|^2$$

$$F_p(q^2) = \frac{1}{4\pi}\int d^3r j_0(qr)\rho_p(r)$$

$$\begin{aligned} ZF_p &= 4\pi\int_0^\infty \rho_p r^2 dr \\ &= \sum_{\nu=1}^{\infty}(-1)^{\nu+1}\frac{4\pi R_p}{q_\nu^2}a_\nu \end{aligned}$$





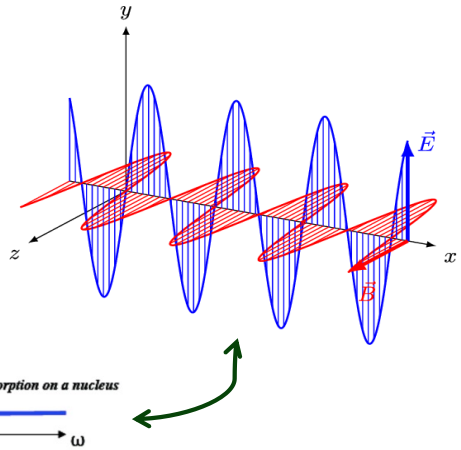


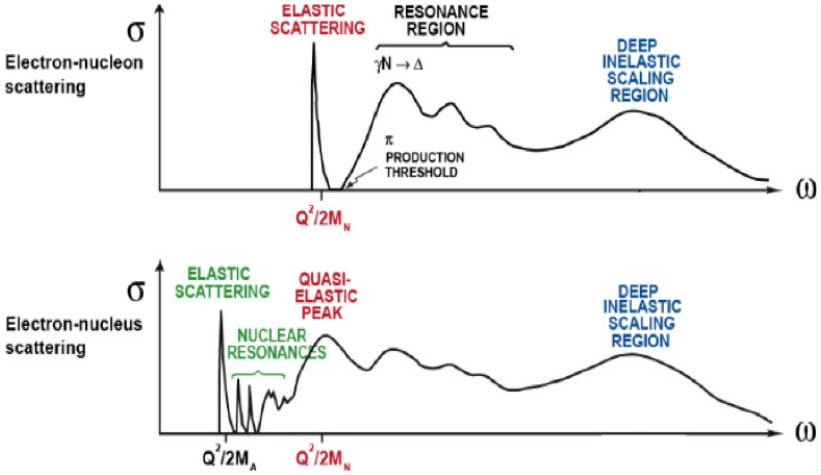
A. Kabir, PhD Thesis, Kent State (2015)

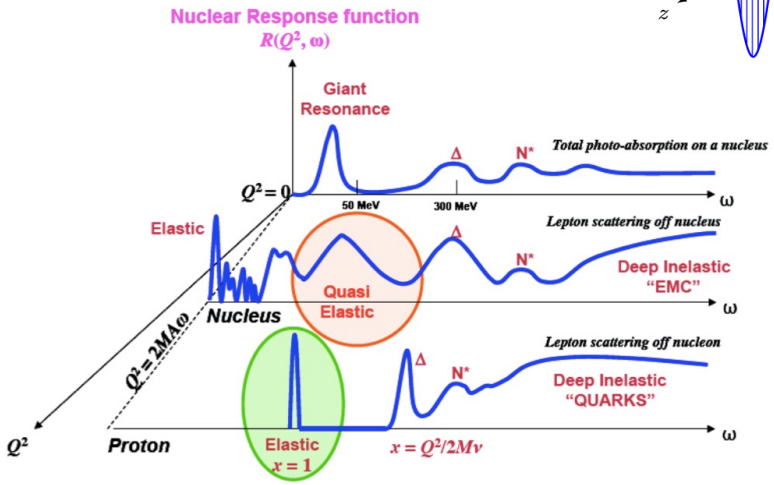
Probing nuclei and nucleons with electrons

The power of (electron) scattering experiments as a microscope
- A [beam] + B [target] = inside view of the nuclear matter!!
- **Invariant mass** technique probing **new particles/states**
- **Missing mass** technique probing **reaction mechanisms**

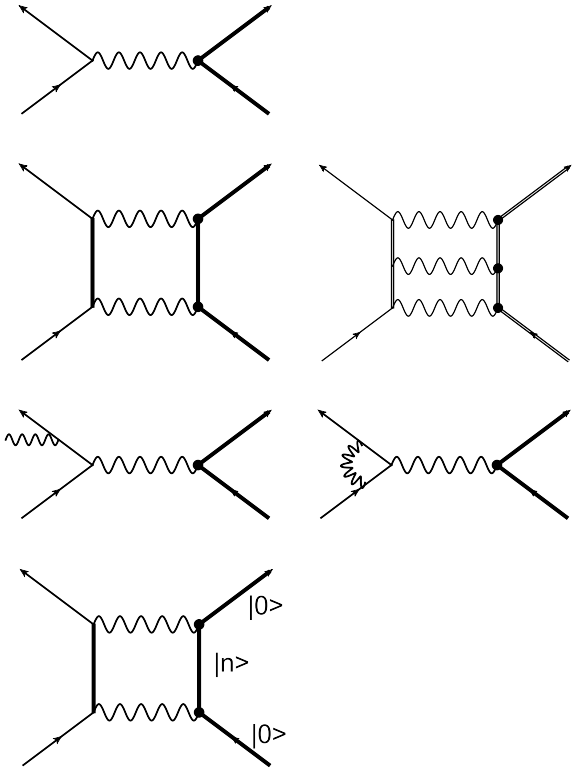
https://commons.wikimedia.org/wiki/File:EM-Wave.gif







Beyond the Born Approximation



Born Approximation

+ ... Coulomb Corrections

partial-wave analysis

+ ... Radiative Corrections

static analysis

+ ... Dispersive Corrections

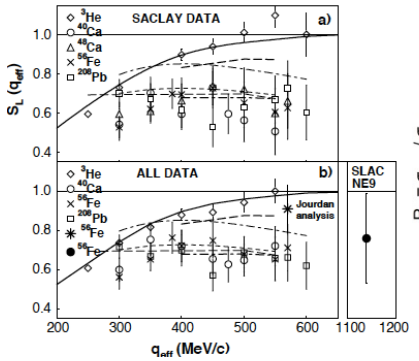
dynamic corrections

What's going on with >1y?

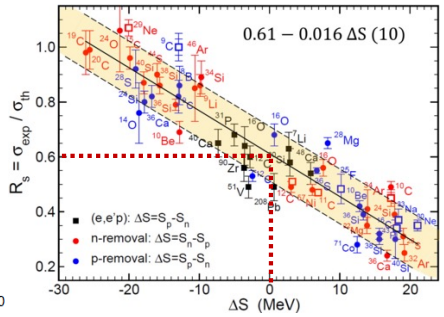
In-medium nucleons & the Coulomb Sum Rule (CSR)

$$S_L(|\mathbf{q}|) = \int_{\omega>0}^{|\mathbf{q}|} \frac{R_L(\omega, |\mathbf{q}|)}{ZG_{E_p}^2(Q^2) + NG_{E_n}^2(Q^2)} d\omega$$

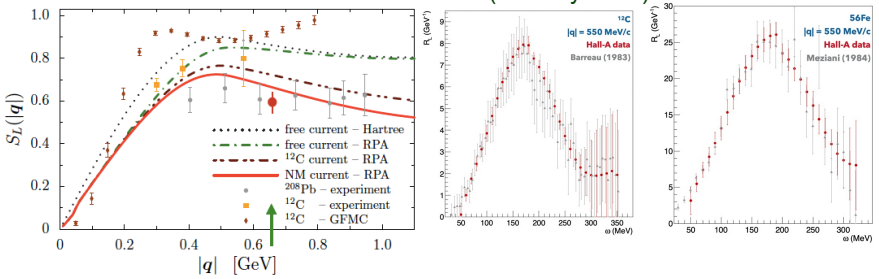
J. Morgenstern & Z. Meziani.
EPJ, **A17**, 451–455, (2003)



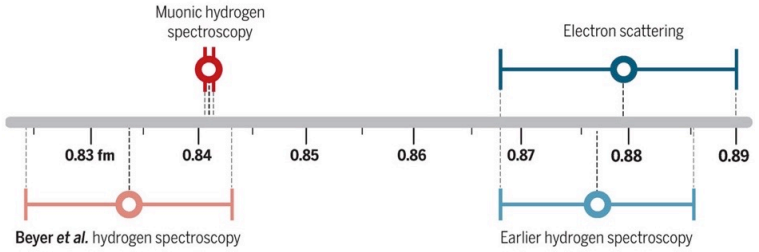
J. A. Tostevin and A. Gade
Phys. Rev. C **103**, 054610 (2021)



M. Paolone, JLab e05-110; CSR quenching?
Hall A/C Collaboration (January 2022)

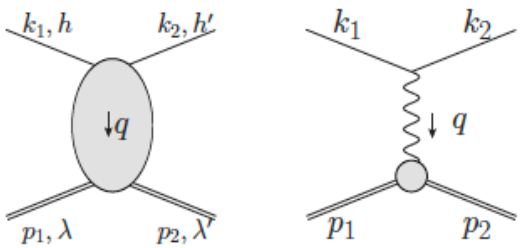


Proton radius



W. Vassen, Science, **358**, 6359, pp. 39-40 (2017)
R. Pohl, Nature, **466** (2010)

G_E/G_M puzzle: the role of two-photon exchange



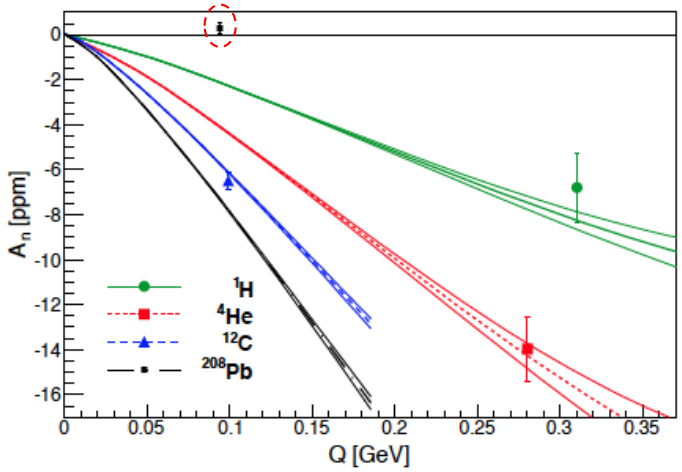
$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{Mott}}{\varepsilon(1+\tau)}\sigma_R$$

$$\sigma_{Mott} = \frac{\alpha^2 \epsilon_2 \cos^2\theta/2}{4\epsilon_1^3 \sin^4\theta/2}$$

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau}G_E^2(Q^2)$$

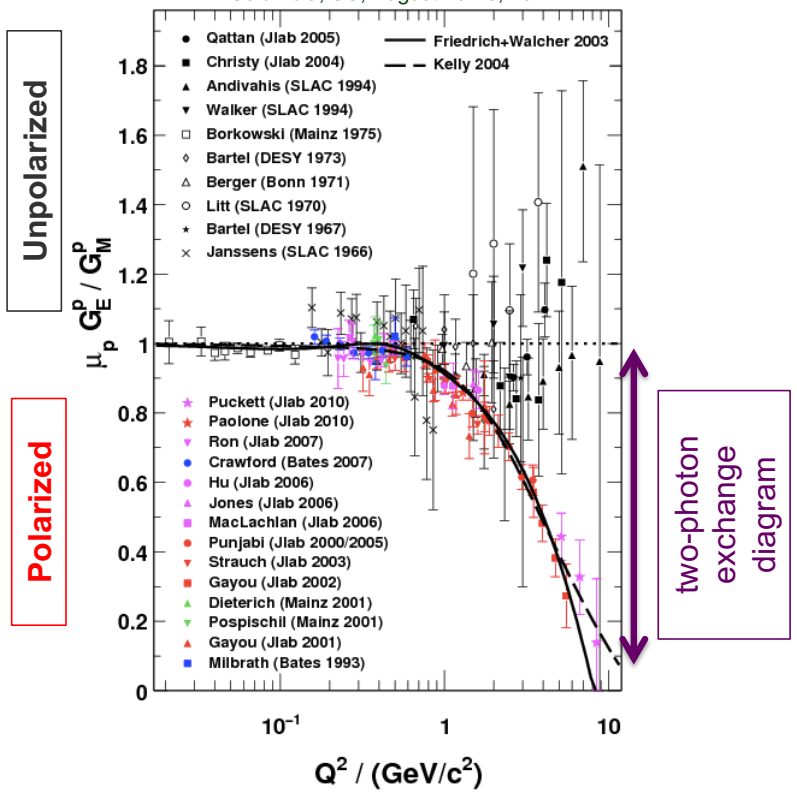
transverse

longitudinal



S. Abrahamyan *et al.* (HAPPEX and PREX Collaborations)
Phys. Rev. Lett. **109**, 192501 (2012)

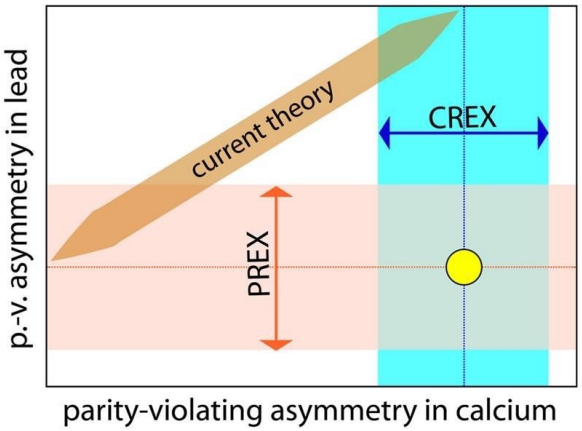
M. Kohl, NSTAR2017
Columbia, SC, August 20-23, 2017



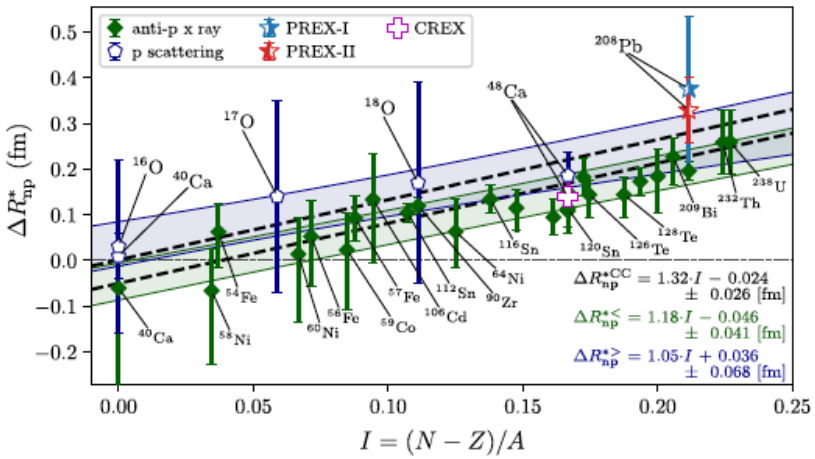
Neutron skin puzzle: the role of virtual excitations [1] (see L. Broussard talk)

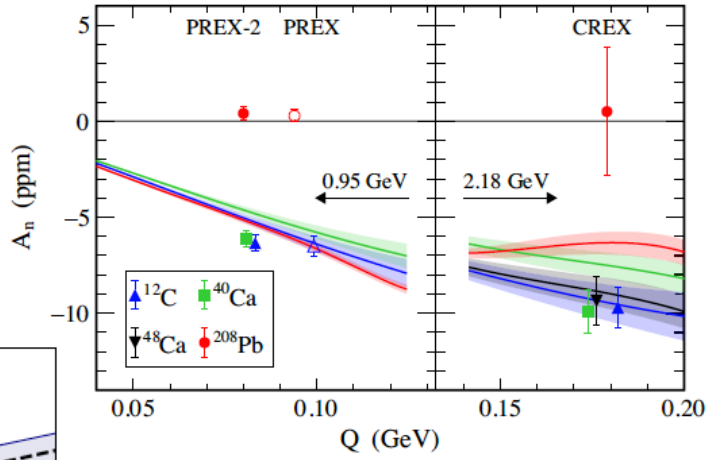
- PREX: D. Adhikari et al., PRL, **126**, 172502 (2021)
  - ^{208}Pb: $R_n - R_p = 0.283 \pm 0.071$ fm
- CREX: D. Adhikari et al., PRL, **129**, 042501 (2022)
  - ^{48}Ca: $R_n - R_p = 0.121 \pm 0.026$ (exp) ± 0.024 (model) fm

P.-G. Reinhard, X. Roca-Maza, and W. Nazarewicz
PRL**129**, 232501 (2022)



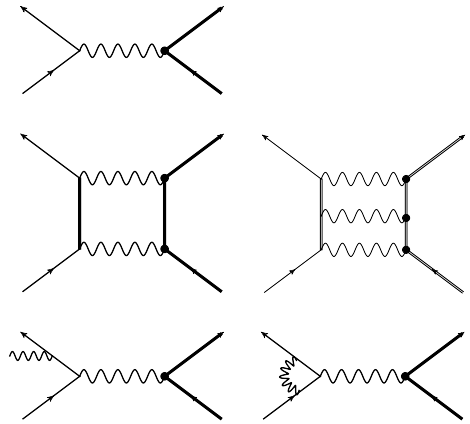
S. J. Novario, D. Lonardoni, S. Gandolfi, and G. Hagen
PRL**130**, 032501 (2023)





Adhikari et al, PRL 128, 142501 (2022)

Neutron skin puzzle: the role of virtual excitations [2]



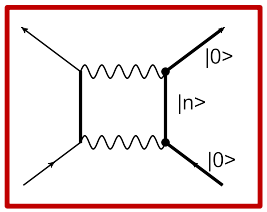
Born Approximation

+ ... Coulomb Corrections

partial-wave analysis

+ ... Radiative Corrections

static analysis



+ ... Dispersive Corrections

dynamic corrections

$$|\mathcal{M}_{elast+disp}|^2 = (\alpha q_e Z)^2 \left[F(q^2)\right]^2 + 2(\alpha q_e Z)^3 \left[F(q^2)\mathcal{R}e\{G(q^2)\}\right] + (\alpha q_e Z)^4 \left[|\mathcal{R}e\{G(q^2)\}|^2 + |\mathcal{I}m\{G(q^2)\}|^2\right]$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm exp} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} \left|F(\mathbf{q}^2)\right|^2$$

Is it what is happening?

Wait … There is More!!
Nuclear Tomography & Polarization?

Meson electro-production

$$
\begin{aligned}
\frac{d\sigma_v}{d\Omega_\eta} &= \frac{|\mathbf{k}|}{k_\gamma^{cm}} P_\alpha P_\beta \{ R_T^{\beta\alpha} + \varepsilon_L R_L^{\beta\alpha} \\
&+ [2\varepsilon_L(1+\varepsilon)]^{1/2} ({}^c R_{TL}^{\beta\alpha} \cos\phi_\eta + {}^s R_{TL}^{\beta\alpha} \sin\phi_\eta) \\
&+ \varepsilon ({}^c R_{TT}^{\beta\alpha} \cos 2\phi_\eta + {}^s R^{\beta\alpha} TT \sin 2\phi_\eta) \\
&+ h[2\varepsilon_L(1-\varepsilon)]^{1/2} ({}^c R_{TL'}^{\beta\alpha} \cos\phi_\eta + {}^s R_{TL'}^{\beta\alpha} \sin\phi_\eta) \\
&+ h(1-\varepsilon^2)^{1/2} R_{TT'}^{\beta\alpha} \}
\end{aligned}
$$

G. Knöchlein, D. Drechsel, L. Tiator
Z. Phys. **A352**, 327-343 (1995)

3D nucleon tomography!!
(DVCs, parton distributions …)

Table 1. Polarization observables in pseudoscalar meson electroproduction. A star denotes a response function which does not vanish but is identical to another response function via a relation in App. A

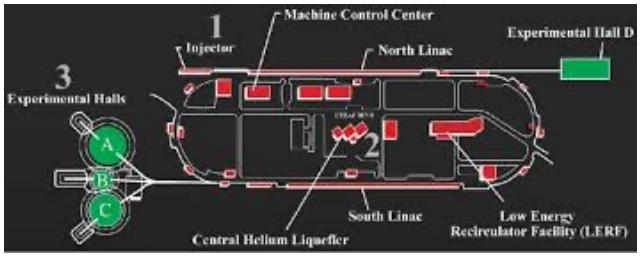
		Target			Recoil			Target + Recoil								
β	–	–	–	–	x'	y'	z'	x'	x'	x'	y'	y'	y'	z'	z'	z'
α	–	x	y	z	–	–	–	x	y	z	x	y	z	x	y	z
T	R_T^{00}	0	R_T^{0y}	0	0	$R_T^{y'0}$	0	$R_T^{x'x}$	0	$R_T^{x'z}$	0	*	0	$R_T^{z'x}$	0	$R_T^{z'z}$
L	R_L	0	R_L^{0y}	0	0	*	0	$R_L^{x'x}$	0	$R_L^{x'z}$	0	*	0	*	0	*
${}^cTL$	${}^cR_{TL}^{00}$	0	${}^cR_{TL}^{0y}$	0	0	*	0	${}^cR_{TL}^{x'x}$	0	*	0	*	0	${}^cR_{TL}^{z'x}$	0	*
${}^sTL$	0	${}^sR_{TL}^{0x}$	0	${}^sR_{TL}^{0z}$	${}^sR_{TL}^{x'0}$	0	${}^sR_{TL}^{z'0}$	0	*	0	*	0	*	0	*	0
${}^cTT$	${}^cR_{TT}^{00}$	0	*	0	0	*	0	*	0	*	0	*	0	*	0	*
${}^sTT$	0	${}^sR_{TT}^{0x}$	0	${}^sR_{TT}^{0z}$	${}^sR_{TT}^{x'0}$	0	${}^sR_{TT}^{z'0}$	0	*	0	*	0	*	0	*	0
${}^cTL'$	0	${}^cR_{TL'}^{0x}$	0	${}^cR_{TL'}^{0z}$	${}^cR_{TL'}^{x'0}$	0	${}^cR_{TL'}^{z'0}$	0	*	0	*	0	*	0	*	0
${}^sTL'$	${}^sR_{TL'}^{00}$	0	${}^sR_{TL'}^{0y}$	0	0	*	0	${}^sR_{TL'}^{x'x}$	0	*	0	*	0	${}^sR_{TL'}^{z'x}$	0	*
TT'	0	$R_{TT'}^{0x}$	0	$R_{TT'}^{0z}$	$R_{TT'}^{x'0}$	0	$R_{TT'}^{z'0}$	0	*	0	*	0	*	0	*	0

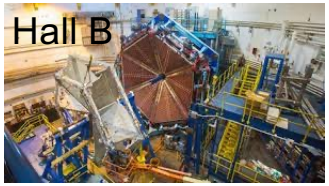
unpolarized

polarized

Thomas Jefferson National Accelerator Facility (Jefferson Lab)

12 GeV, 14 kW
Polarized e^-
CW (1.5 GHz)

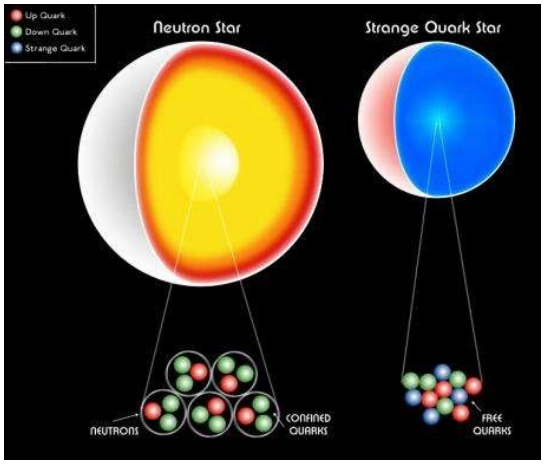


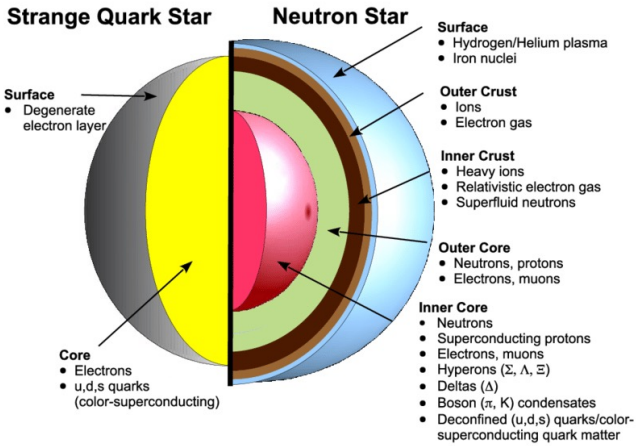


Neutron Stars And Strange Stars? (see P. Giuliani, B. Ramson, B. Dongwi talks)

- **Need to understand elementary process**
- Start with lightest ones
  - uds: $\Lambda(1115)$, $\Sigma^0(1192)$
  - uus: $\Sigma^+(1189)$
  - dds: $\Sigma^0(1197)$
- Formation of hypernuclei: ${}^{A}X \rightarrow {}^{A-1}X_{Y=[\Lambda,\Sigma]}$
  - Interactions: NN, ΛN, ΣN, ΛNN …

F. Weber et al., Mod. Phys. Lett. A **29**, 23 (2014)





Homework

- Calculate the elastic scattering kinematics (use relativistic kinematics)
  - Known : incident electrons (E_{inc}, $\mathbf{P}_{inc}$), scattering angle (Θ)
  - Unknown : outgoing electrons (E_{out}, $\mathbf{P}_{out}$)
- Assuming a homogenous sphere of radius R
  - Known density

$$\rho(r) = \begin{cases} \frac{3}{4}\pi R^3 & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases}$$

  - Replacing the density $\rho(r)$ to show that the form factor $F(\mathbf{q}^2)$ is

$$F(\mathbf{q}^2) = 4\pi \int \rho(r) \frac{\sin(|\mathbf{q}|r/\hbar)}{|\mathbf{q}|r/\hbar} r^2 dr = \frac{3}{\alpha^3}(\sin\alpha - \alpha\cos\alpha)\ ; \alpha = |\mathbf{q}|R/\hbar$$